EXAM 2 SOLUTIONS

Finance 34600 – Investment Theory

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INSTRUCTIONS:

1. You have 75 minutes to complete the exam.
2. The exam is worth a total of 100 points.
3. You may use a calculator and an 8 ½ by 11 inch formula sheet. You must hand in the formula sheet with your exam (put your name on it).
4. Allocate your time wisely. Use the number of points assigned to each problem as your guide.
5. In order to get full credit on the problems, you must show ALL your work!

Multiple Choice (32 points)
Choose the best answer for each of the following questions. The questions are worth 4 points each.

1. The expected return on the market is 15% and the risk-free rate is 5%. The expected return on security XYZ is 25%. Assuming CAPM holds and the market is in equilibrium, what must be the Beta on security XYZ?

   a) 1.50
   b) 1.67
   c) 2.00
   d) 2.50

2. You estimate a market model regression for Intel stock. Based on your estimates, the Beta of Intel stock is 1.2. You also estimate that the standard deviation of returns is 35% for Intel and 21% for the market. What proportion of Intel's variance is systematic (market) risk?

   a) 60.0%
   b) 86.4%
   c) 36.0%
   d) 51.8%

3. In a world with no risk-free security, the optimal risky portfolio is determined by:

   a) The point of tangency between the investor's indifference curves and the CAL
   b) The point of tangency between the efficient frontier and the investor's indifference curves
   c) The point of tangency between the efficient frontier and the CAL
   d) The minimum variance point on the efficient frontier

4. Consider a portfolio formed by placing a portion of your investment in risky security A and the rest in risky security B, where the correlation between the risky securities is 0.1. Which of the following statements regarding the standard deviation of your new portfolio is correct?

   a) The portfolio standard deviation will be less than the weighted average of the individual security standard deviations.
   b) The portfolio standard deviation will be equal to the weighted average of the individual security standard deviations.
   c) The portfolio standard deviation will be greater than the weighted average of the individual security standard deviations.
   d) The answer cannot be determined from the information provided.

5. Your boss at *TimeTheMarket Hedge Fund* asks you to analyze the market timing performance of the fund. Based on what you learned in Professor Corwin's class, you decide to estimate the Hendrikkson-Merton model of market timing. You find that your hedge fund has a beta of 1.4 on days when the market return is negative and a beta of 0.95 on days when the market return is positive. Which of the following statements correctly describes your fund's market timing ability?

a) Your firm has good market timing skills
b) Your firm has no market timing skills
c) Your firm has bad market timing skills
d) Your firm's market timing skills cannot be determined from this analysis

6. You are evaluating average returns for an investment manager over the past two years. At the beginning of the first year, the manager invested $1 million in the S&P 500 index. At the beginning of the second year, the manager added another $1 million to his investment in the S&P. The S&P returned 15% during the first year and 8% during the second year. What is the manager's dollar-weighted average return? (Assume no dividends were paid.)

a) 10.375% (this is the only possible answer, since it must be less than 11.5%)
b) 11.500%
c) 12.125%
d) 12.975%

7. Security A has an expected return of 9% and a standard deviation of 11%. Security B has an expected return of 14% and a standard deviation of 19%. Calculate the weights in Security A and Security B that would give the lowest variance among all possible combinations of these two securities. Assume that the correlation between the two securities is zero.

a) 25.1% in A, 74.9% in B
b) 63.3% in A, 36.7% in B
c) 36.7% in A, 63.3% in B
d) 74.9% in A, 25.1% in B

8. You wish to compare the performance of a small growth fund to that of the Nasdaq index. The growth fund has an average return of 14% and a standard deviation of 17%. The Nasdaq index has an average return of 20% and a standard deviation of 23%. If the risk-free rate is 2%, what is the M^2 measure for this portfolio (compared to the Nasdaq benchmark)?

a) -1.765%
b) 2.015%
c) -9.132%
d) 3.320%

Problems (68 points)
Answer each of the questions below completely. Show all your work.

9. **Probability of Loss (10 points)**

Your client is considering shifting a large portfolio of their retirement funds into a market index fund. Over the next year, you expect this fund to have an expected return of 13% and a standard deviation of 26%. Your client likes to think about risk in terms of the likelihood of potential losses. Assuming that returns follow a Normal distribution, perform the necessary calculations to complete the statement below. A Normal distribution table is provided on the last page of the exam.

There is a 10% chance that your portfolio will earn an annual return of _____ % or lower.

The solution here is solved in the reverse order of those we did in class. In class, I gave your R* and asked you to solve for the probability. Here, I give you the probability and ask you to solve for R*. Note that in order to solve the problem, you need to recognize that the negative side of the standard Normal distribution is symmetric with the right.

You want to know the return at which there is a 10% chance of earning less. To solve for this point, note that you can first identify the Z value at which there is a 10% chance of earning *more*. According to the table, there is a 39.97% chance of getting a Z between 0 and +1.28. The inverse must then also be true, there is a 40% chance of getting a Z between 0 and -1.28, or a 10% chance of getting a Z less than -1.28. Setting Z equal to -1.28, we get:

$$Z = -1.28 = \frac{R^* - .13}{.26}$$

$$R^* = -1.28(.26) + .13 = -20.3\%$$

10. **CAPM Implications (10 points)**

Explain whether the following data is possible or not possible assuming CAPM holds and the economy is in equilibrium. Show any necessary calculations to support your answer.

	E(R)	β	σ
Stock X	15.1%	1.45	32.0
Market	11.5%	1.00	22.0
Risk-free	3.5%	0.00	0.0

Rew-to-Var$_{Mkt}$ = (.115-.035)/.22 = 0.3636
Rew-to-Var$_X$ = (.151-.035)/.32 = 0.3625

α_X = 0.151 – (.035 + 1.45(.115-.035)) = 0.151 – 0.151 = 0

The security is on the SML (alpha = 0) and below the CML, so the data are consistent with CAPM.

11. **Arbitrage Pricing Theory (12 points)**

Assume that expected returns in the economy are related to two risk factors. An analyst at your firm has provided you with the data below related to the expected returns and risk-factor sensitivities (betas) of two well-diversified mutual funds. Assuming this data is correct, solve for the APT pricing equation in this economy. Write the APT pricing equation that you could use to solve for the expected return on any portfolio. (Note: If you cannot solve this problem write out the general form of the APT equation with two risk factors.)

	E(R)	β_{Factor1}	β_{Factor2}
Fund 1	18.0%	-1.4	2.3
Fund 2	11.0%	0.0	0.9
Risk-free	2.0%	0.0	0.0

$$E(R) - R_f = \beta_{F1}(E(R_{F1}) - R_f) + \beta_{F2}(E(R_{F2}) - R_f) = \beta_{F1}(\gamma_1) + \beta_{F2}(\gamma_2)$$

$$(1) \therefore \longrightarrow 0.18 - 0.02 = -1.4\gamma_1 + 2.3\gamma_2$$

$$(2) \longrightarrow 0.11 - 0.02 = 0\gamma_1 + 0.9\gamma_2 \longrightarrow \gamma_2 = \frac{.11 - .02}{.9} = 0.10 = 10.0\%$$

$$0.18 - .02 = -1.4\gamma_1 + 2.3(0.10) \longrightarrow \gamma_1 = \frac{.16 - .23}{1.4} = 0.05 = 5.0\%$$

$$E(R) = 2.0\% + \beta_{F1}(5.0\%) + \beta_{F2}(10.0\%)$$

12. **Risk Preferences and Capital Allocation (14 points)**

You work for a money manager and are trying to convince a new client that they should switch from their current fund (*Bond Fund*) to a combined stock and bond fund (*Combined Fund*). You have the following data on the two funds and the risk-free security.

Fund	Expected Return	Standard Deviation
Bond Fund	8.0%	11.0%
Combined Fund	13.0%	17.0%
Risk-free Security	4.0%	0.0%

a) (5 points) The client informs you that he currently hold 80% of his portfolio in the *Bond Fund* and 20% in the risk-free security. He explains that this is his optimal mix of the two securities. Based on the investor's current holdings, calculate the investor's risk aversion level.

$$w^* = 0.8 = \frac{0.08 - 0.04}{A(.11)^2} \longrightarrow A = \frac{0.08 - 0.04}{0.8(.11)^2} = 4.1322$$

b) (5 points) Using the risk aversion level you calculated in part (a), calculate the investor's optimal portfolio weights in the *Combined Fund* and the risk-free security. (Note: if you cannot solve part (a), answer this question assuming a risk aversion level of A=2.)

$$w^*_{CF} = \frac{0.13 - 0.04}{4.1322(.17)^2} = 0.7536 = 75.36\%$$

$$w_{R_f} = 1 - .7536 = 24.64\%$$

c) (4 points) Your client says that he is nervous about switching to the new fund due to his high aversion to risk. In one or two sentences, explain to your client why the new portfolio mix may be better for him even given his risk aversion.

The new fund offers a better tradeoff between risk and return. In other words, at ANY level of risk, you can get higher expected return using the new fund and at ANY level of return, you can get lower risk using the new fund. This allows the investor to get to a higher level of utility.

13. **Portfolio Theory (22 points)**

A portfolio of U.S. Stocks has an expected return of 11% and a standard deviation of 24%. A portfolio of international stocks has an expected return of 19% and a standard deviation of 32%. The correlation between U.S. and international stocks is 0.3 and the risk-free rate is 4%.

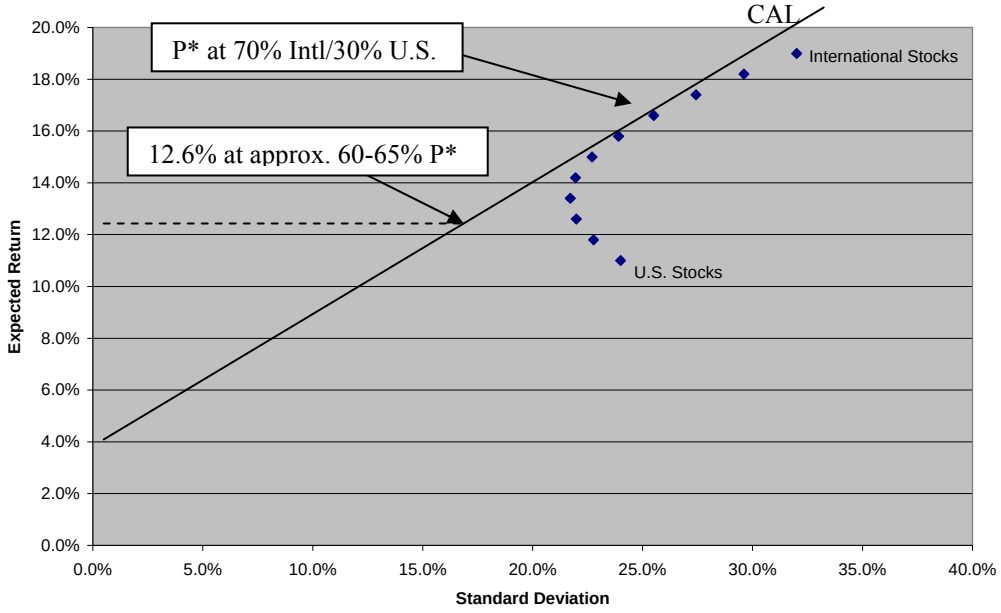
a) (12 points) Consider the possible portfolios you could create combining U.S. and international stocks. Calculate the expected return and standard deviation on a portfolio that invests 80% in U.S. Stocks and 20% in international stocks.

$$Rule1^{*}: E(R) = .8(.11) + .2(.19) = 12.6\%$$

$$Rule2^{*}: \sigma^2 = (.8^2)(.24^2) + (.2^2)(.32^2) + 2(.8)(.2)(.24)(.32)(.3) = .0483328$$

$$\longrightarrow \sigma = \sqrt{.0483328} = 21.985\%$$

b) (6 points) The chart below shows the various portfolios formed by combining U.S. Stocks and International Stocks with weights varying in 10% increments from 100% U.S. Stocks to 100% International Stocks. Draw the approximate CAL for the optimal combination of U.S. Stocks and International Stocks and label the optimal portfolio on the graph. What are the approximate weights in the optimal portfolio?



c) (4 points) Identify the point on the CAL that earns the same expected return as the portfolio described in part (a) above. What are the approximate weights in the risk-free and the combined portfolio required to achieve this return? (note: no calculations are necessary here.)

You earn 12.6% at a point on the CAL where the weight in P* is approximately 60-65% and the weight in the risk-free is approximately 35-40%.

Normal Distribution Table:
Area Under the Normal Curve from 0 to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3079	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441
1.6	0.4452	0.4463	0.4474	0.4485	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4700	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4762	0.4767
2.0	0.4773	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817